Stocks Green Primary School

Progression in Written Calculation

Introduction

This whole school approach to written methods of calculation outlines the route that we expect most pupils at this school to take in developing their understanding of calculation.

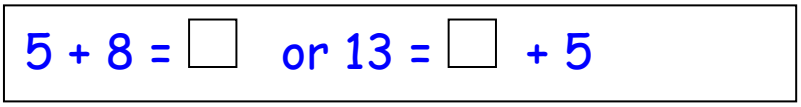
The Early Stages of Calculation

The ability to calculate mentally, supported by informal jottings, is an essential prerequisite for progression to written calculation. In the early years, children will use oral methods: developing number recognition and simple counting one by one, leading to an understanding of the sequence and order of numbers (first, second, third etc). More sophisticated mental counting strategies follow this —

- counting objects (how many)
- counting forwards and backwards
- counting on from a given number (games like 'snakes and ladders', 'Ludo' etc support this well)
- counting in units, twos and tens.

The use of games, number lines, number squares, counting stories and songs and other visual aids support this, along with a variety of 'real life' situations to put the maths in context (role play - shops etc.) Children are encouraged to learn and recall simple number facts and use appropriate vocabulary when talking about mathematics — an important stage in securing their understanding of calculation strategies and problem solving skills.

Alongside developing their mental/oral skills, written recording takes place; first learning to read, interpret and complete statements in horizontal form:



More informal jottings are also made by the children, which may not be easy for someone else to follow, but are an important stage in clarifying mental processes and getting the right answer.

An understanding of place value and inverse operations is necessary in order that children are able to judge the accuracy of their answers and for this reason, addition is taught with reference to subtraction in Key stage 1 and multiplication with division in Key stage 2.

The move from part written/part mental calculation to using standard written methods cannot take place until a child is able to judge whether an answer is reasonable, therefore this is not rushed or taught at a prescribed age. These methods are introduced when a child is ready, through differentiated teaching towards the end of Key Stage 1 — however it must be stressed that **some children may not meet this until a later time**. An indicator of readiness to move onto written methods is when a child can add or subtract two 2-digit numbers mentally.

Please note -

The stages in this document are stages in development, not year groups or key stages.

Addition

Stage 1

Children need a secure understanding of **partitioning and recombining** numbers, so, at this stage, mental methods using partitioning supported by the use of the number line are used.

Partition and recombine by breaking units of 6, 7, 8 or 9 into '5 and a bit'

e.g. mentally work out and explain that –

5 + 8 = 5 plus (5 and 3)
= 5 + 5 + 3
= 10 + 3
= 13

Or

9 + 8 = (5 + 4) plus (5 + 3)
= 5 + 5 + 4 + 3
= 10 + 7
= 17

Partition into tens and units

e.g. mentally work out and explain that —

12 + 23 = (10 + 2) plus (20 + 3)
= (10 + 20) + (2 + 3)
= 30 + 5
= 35

Or

12 + 23 = 12 + 20 + 3
= 32 + 3
= 35

Or

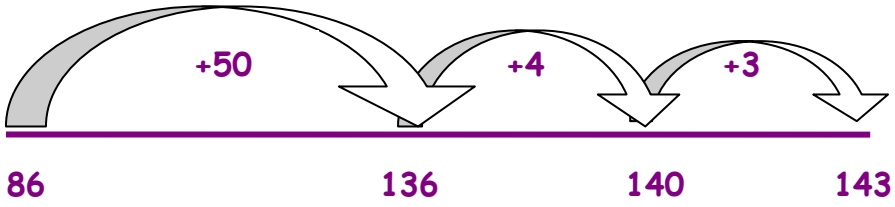
47 + 32 = 79
40 + 30 = 70
7 + 2 = 9
= 79

Stage 2

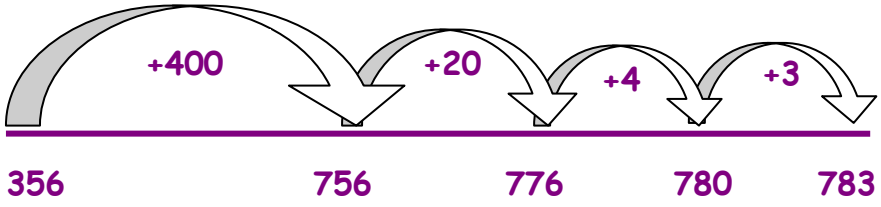
Extend the stage 1 methods to bigger numbers and continue using the number line to count on in multiples of 1, 10 and 100.

Counting on in multiples of 100, 10 or 1

86 + 57 = 86 + 50 + 7 = 136 + 7 = 143



356 + 427 = 356 + (400 + 20 + 7)
= 756 + 20 + 7
= 776 + 7
= 783



The children will begin to record calculations in preparation for an efficient standard method. They will know that units line up under units, tens under tens and so on.

They will learn to use an expanded vertical layout — adding the least significant digit first in preparation for 'carrying' later on. (Although they will also be taught that addition is commutative.)

e.g

```
  76
+ 47
  13   ← least significant digit first (units first)
 110
 123
```

In order to support mental methods of adding on (it is easier to count on a smaller number) and the layout of written forms of subtraction, the children are encouraged to put the bigger number on the top in this form of written method.

Stage 3

Extend the expanded vertical layout to bigger numbers and encourage the use of **estimation** first.

e.g. 493 + 368 Estimate: 370 + 500 = 870

```
  493
+ 368
  11
 150
 700
 861
```

This method then progresses to the use of '**carrying**' below the line

```
  625        783        367
+ 48       + 42       + 85
 673        825        452
  1          1        1 1
```

This then extends to the addition of sums of money (**decimals**). The children are taught that decimal points should line up under each other, particularly when adding or subtracting mixed amounts.

e.g. £3.59 + 78p

```
 3.59
+ .78
 4.37
 1  1
```

Always refer to each digit by its proper place value e.g. in 953 the 5 is 50.

Stage 4

Continue to develop the methods taught in stage 3 and extend to using numbers with at least four digits and 'carrying' 2 or more numbers, also encouraging estimation.

Develop this further to column addition of several numbers with different numbers of digits.

e.g. 58 + 671 + 9 + 468 + 2187

```
     2 1 8 7
       6 7 1
       4 6 8
         5 8
+          9
   ---------
   3 3 9 3
   1 2 3
```

If children are showing misconceptions at any stage they must refer back to the previous strategy.

At this stage, children will also be making increasing use of the calculator.

Subtraction

Stage1

Subtraction is taught in relation to addition (inverse) developing the children's knowledge and understanding of:

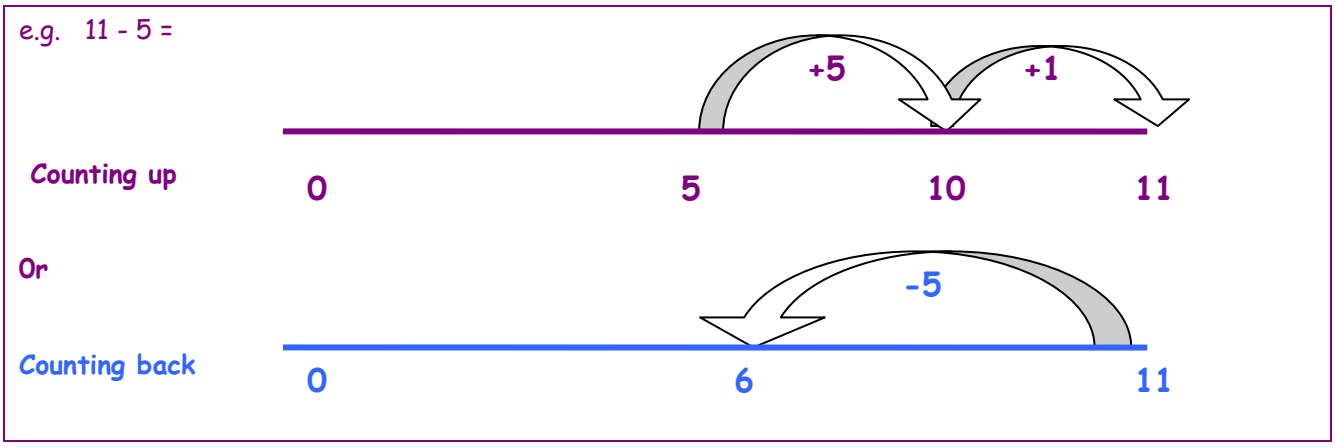
- Taking away
- Finding the difference between
- How many more make... (complementary addition)

The children will learn 'by heart' number bonds for addition/subtraction for numbers up to 10 and then to 20, and understand the concept of doubling and halving.

They will be taught to respond rapidly to questions phrased in a variety of ways:

- 4 take away 2
- take 2 from 7
- 7 subtract 3
- double 4
- half of 6
- two fives
- how many less than 6 is 4 etc,

Oral questioning and mental subtraction is supported by the use of the empty number line and the hundred square.



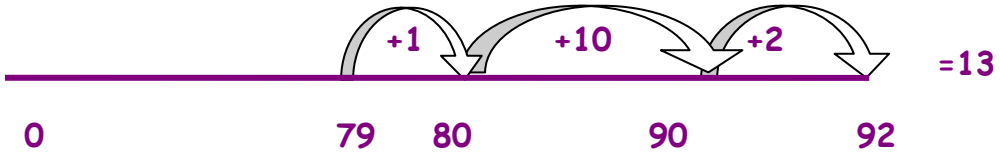
The children begin to understand the principle that subtraction reverses addition (inverse operation) and that subtraction is not commutative.

e.g. 4 - 2 is different from 2 - 4.

Stage 2

Consolidating and building upon stage 1 — counting up in multiples of ten and then units to get the target number

e.g. 92 – 79 =



1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

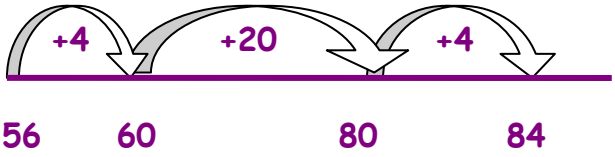
The use of the hundred square relates to the empty number line and is another way of modelling it.

Stage 3

Pupils develop the use of informal jottings to support mental methods used.

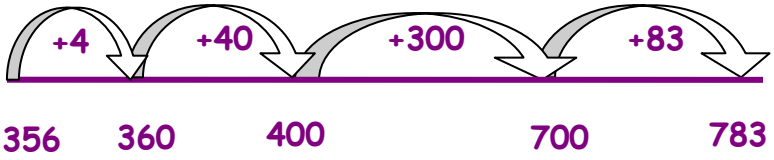
(A) Counting up from the smaller to the larger number (complementary addition)

e.g. 84 -56 56 + 4 + 20 + 4 = 84



56
4
60
20
80
4
84
28

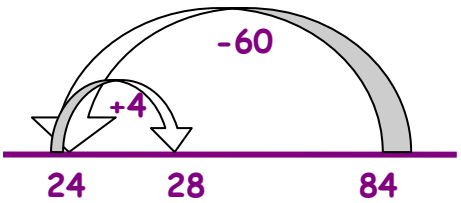
783 – 356



783
- 356
4 to 360
40 to 400
300 to 700
83 to 783
427

(B) Compensation (take too much, add back)

84 – 56 = 84 – 60 + 4
= 24 + 4
= 28



783 – 356 = 783 – 400 + 44
= 383 + 44
= 427

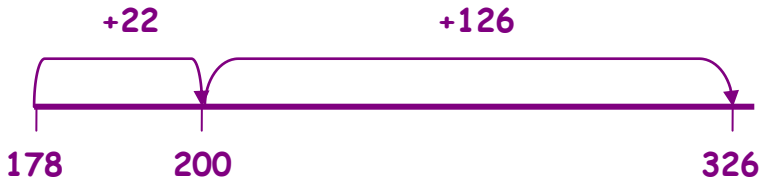
783
- 356
383 take 400
+ 44 add 44
300
120
7
427

(C) Counting back. e.g. for 84 – 56, show jottings or draw arrows on a number line to show 84 – 10 – 10 – 10 – 10 – 10 – 6 = 56. Then progress to shorter forms e.g. 84 – 56 = 84 – 20 – 20 – 10 – 6 = 56.

Stage 4

Still using the number line principle, but reducing the number of stages further by using their knowledge of pairs that total 100.

```
  326
- 178
   22   (→ 200)
 +126   (→ 326)
  148
```



Stage 5

Pupils will leave counting up in favour of decomposition - except those who still need support in previous methods, or who find it easier to conceive of subtraction as counting on.

Standard written methods

Continue to develop an efficient standard method that can be applied generally. e.g.

(C) Decomposition

```
  754  =  700  +  50  +  4
- 286     200  +  80  +  6        leading to

       =  700  +  40  +  14        744
          200  +  80  +  6      - 286

       =  600  +  140  +  14      644        754
          200  +  80   +  6      - 286      - 286
          400  +  60   +  8        468        468
```

Multiplication

Stage 1

The children will develop counting experiences that lead to repeated addition; hundred square activities prepare children for multiplication in year 2.

Stage 2

Children will reinforce their understanding of repeated addition

e.g. 5 x 3 = 5 + 5 + 5 **or** 3 lots of 5 **or** 3 times 5 **or** 5 x 3 (**or** 3 x 5)

They will also be taught that doubling reverses halving (inverse operations)

e.g

11 x 2 = 22 implies that half of 22 is 11

or 22 ÷ 11 = 2

Learning Multiplication facts

For the next stages, children need to have learnt and can recall multiplication facts (up to 10 x 10).

Children should first learn the facts for multiplying by 2,4, 5 and 10 then progress onto those for multiplying by 3,6,7,8 and 9.

Stage 3

Number lines, hundred squares and other visual resources are used to build up mental methods of multiplication using partitioning.

e.g. 38 x 7 = (30 x 7) + (8 x 7)

For able pupils, who are secure in previous methods taught, the grid method of multiplication (which uses partitioning) is introduced.

Grid layout, expanded working

X	30	8	
7	210	56	266

Extended to bigger numbers.

Example: 56 x 27. Estimate: 1800 because 60 x 30 =1800

56 x 27 = (50 + 6) x (20 + 7)

X	50	6	
20	1000	120	1120
7	350	42	392
			1512

Stage 4

The grid method extends to bigger numbers and decimals.

Example: 23.5 x 12. Estimate: 25 x 10 = 250

23.5 x 12 = (20 + 3 + 0.5) x (10 +2)

X	20	3	0.5	
10	200	30	5	235
2	40	6	1	47
				282

Stage 5

Some children may still need to use the grid method. However, most children will progress to a vertical format with an expanded layout. All vertical layouts are taught alongside the grid method to ensure understanding

Vertical format, expanded working.

Again, start with a calculation the children can do mentally. Show links with the grid method.

Link to Grid method:

```
  38
X  7
 210   (30 X 7 = 210)
  56    (8 X 7 =  56)
 266
```

X	30	8	
7	210	56	266

Link to Grid method:

```
  56
X 27
1000   (50 X 20 = 1000)
 120    (6 X 20 =  120)
 350    (50 X  7 =  350)
  42     (6 X  7 =   42)
1512
 1
```

X	50	6	
20	1000	120	1120
7	350	42	392
			1512

Stage 6

In year 6, standard methods will be developed into working with larger numbers and decimals, approximating first. Where calculations are set out in columns, the children must know that units line up under units, tens under tens and so on. All vertical methods link to the grid method.

```
  38
X  7
 266
  5
```

Link to grid method:

(Children may need to use jottings to support the individual multiplications in this method.)

```
  56
X 27
1120   (56 X 20 )
 392   (56 X  7)
1512
  1
```

X	50	6	
20	1000	120	1120
7	350	42	392
			1512

This extends to the more compact version of the vertical method of multiplication.

Standard written methods

Continue to develop an efficient standard method that can be applied generally, approximating first.

Short multiplication: ThHTU x U

4346 x 8 is approx. 4500 x 10 = 45000

```
  4346
X    8
4000 X 8   32000
 300 X 8    2400
  40 X 8     320
   6 X 8      48
           34768
```

Long multiplication: HTU x TU

352 x 27 is approx. 350 x 30 = 10500

```
   352
x   27
352 X 20   7040
352 X  7   2464
           9504
             1
```

Extend to decimals with up to two decimal places

Multiply by a single digit, approximating first. Know that decimal points should line up under each other.

4.92 X 3 is about 5 x 3 = 15

4.92 x 3	4.00 x 3	=	12.00
	0.90 x 3	=	2.70
	0.02 x 3	=	0.06
			14.76

Division

In Key Stage 1 the children will have plenty of practice counting in 2's and 10's forwards and backwards, sharing and grouping objects and numbers. Links are made to the inverse (division/multiplication) and division expressed as repeated subtraction. To support this, the number line is used as a visual cue. The language of grouping and sharing is emphasised.

By the end of Key stage 1, children should experience, sharing and grouping objects where there is a **remainder** at the end. It is very important that this is in practical contexts. This is a good starting point for further understanding of remainders in Key Stage 2.

As children progress into Key Stage 2, children should experience grouping into given numbers, leading to repeated subtraction to reinforce their understanding.

4 x 3 = 12

3 X 4 = 12

2 x 6 = 12

Six groups of 2

Learning Division facts

Also, children need to have learnt and can recall the division facts which correspond with matching multiplication facts (up to 10 x 10).

e.g. learn that 54 divided by 9 = 6 because they know that 6 x 9 = 54.

Children should first learn division facts for dividing by 2,4, 5 and 10 then progress onto those for dividing by 3,6,7,8 and 9.

Stage 1

Children are taught to recognise the usefulness of subtracting multiples of the divisor. In this stage, children need to understand the concept of **remainders**.

72 ÷ 5	72	
	- 50	5 x **10**
	22	
	- 20	5 x **4**
	2	

Answer : **14** remainder 2

Stage 2

Stage 1 methods are now applied to larger numbers, estimating first. This is often called 'chunking'.

6) 196			6) 196		
- 60	6 x **10**		- 120	6 x **20**	
136			176		
- 60	6 x **10**		- 60	6 x **10**	
76			16		
- 60	6 x **10**		- 12	6 x **2**	Contract as children's mental strategies improve
16			4		
- 12	6 x **2**				
4					

Answer 32 R 4

Stage 3

By year 6, most children would be expected to be working on long division HTU ÷ U, although less able pupils may still need support in an expanded method using convenient multiples.

Long Division: HTU ÷ TU

Approximate first. 972 ÷ 36 is approximately 1000 ÷ 40 = 25.

```
36 ) 972
   - 720        36 x 20
     252
   - 252        36 x 7
       0
```

Answer : **27**

Extend to decimals with up to two decimal places

Approximate first. Know that decimal points should line up under each other.

87.5 ÷ 7 is approximately 80 ÷ 8 = 10.

```
7 ) 87.5
  - 70.0      7 x 10
    17.5
  - 14.0      7 x  2
     3.5
  -  3.5      7 x  0.5      Answer : 12.5
     0.0
```